Multiuser CDMA Scenario.

Two users.

User 0 — symbol a_0 — code $C_0(n)$

User 1 — symbol a_1 — code $C_1(n)$.

Transmitted symbol $x(n)$

$$x(n) = \underbrace{a_0 c_0(n)}_{\text{sym 0 x code 0}} + \underbrace{a_1 c_1(n)}_{\text{sym 1 x code 1}}$$

Transmitted Multiuser CDMA signal.

Received Signal

$$y(n) = h\,x(n) + W(n)$$

$W(n)$: IID Gaussian noise samples, mean $= 0$, variance $= \sigma^2$

h: Fading coefficient

$$y(n) = h\underbrace{\left(a_0 C_0(n) + a_1 C_1(n)\right)}_{x(n)} + W(n)$$

$$= h\,a_0 C_0(n) + h\,a_1 C_1(n) + W(n).$$

At the receiver, correlate with code of user 0,

$$\frac{1}{N}\sum_n y(n)\,c_0(n)$$

$$= \frac{1}{N}\sum_n \left(h\,a_0\,c_0(n) + h\,a_1\,c_1(n)\right)c_0(n)$$

$$+ \frac{1}{N}\sum_n W(n)\,c_0(n)$$

W = Noise component

mean = 0

variance = σ^2/N.

$$y(n) = h a_0 \frac{1}{N} \sum_n c_0^2(n) + h a_1 \frac{1}{N} \sum_n c_0(n) c_1(n) + \tilde{w}$$

$$\frac{1}{N} \sum_n c_0^2(n) = \frac{1}{N} \sum_n 1 = \frac{1}{N} \times N = 1$$

$$y(n) = h a_0 + h a_1 \frac{1}{N} \sum_n c_0(n) c_1(n) + \tilde{w}$$

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Signal

Multiuser Interference component (MUI)

Noise

arises because of simultaneous transmission to multiple users.

$$r_{01} = \frac{1}{N} \sum_n c_0(n)\, c_1(n)$$

Each $c_0(n), c_1(n) = \pm 1$ with prob $\frac{1}{2}$ each.

$$E\{c_0(n)\} = \frac{1}{2} \times (+1) + \frac{1}{2}(-1)$$

$$E\{c_1(n)\} = 0$$

Further, $C_0(n)$, $C_1(n)$ of user 0 and user 1 respectively can be assumed to be independent.

$$E\{C_0(n)\,C_1(n)\}$$
$$= E\{C_0(n)\} \cdot E\{C_1(n)\}$$
$$= 0 \times 0 = 0.$$

$$E\{r_{01}\} = E\left\{\frac{1}{N}\sum_n c_0(n)\,c_1(n)\right\}$$

$$= \frac{1}{N}\sum_n E\{c_0(n)\,c_1(n)\}$$

$$= \frac{1}{N}\sum_n 0 = 0.$$

Power,

$$E\{|r_{01}|^2\}$$

$$= E\left\{ \frac{1}{N} \sum_n C_0(n)\, G_1(n) \times \frac{1}{N} \sum_m C_0(m)\, G_1(m) \right\}$$

$$= E\left\{ \frac{1}{N^2} \sum_n \sum_m C_0(n)\, G_1(n)\, C_0(m) G_1(m) \right\}$$

$$= \frac{1}{N^2} \sum_n \sum_m E\{C_0(n) C_0(m)\} \times E\{G(n) G(m)\}$$

Since $C_0(n)$, $G(m)$ are PN Sequences,

$$E\{C_0(n) C_0(m)\} = E\{C_0(n)\} E\{C_0(m)\}$$
$$= 0 \times 0 = 0 \quad \text{if } n \neq m$$

If $n = m$,

$$E\{c_0(n)\, c_0(m)\} = E\{c_0^2(n)\}$$

$$= E\{1\} = 1$$

$$E\{c_0(n)\, c_0(m)\} = E\{G(n)\, G(m)\} = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases}$$

$$E\{r_{01}^2\} = \frac{1}{N^2}\sum_n\sum_m E\{c_0(n)c_0(m)\} \times E\{g(n)g(m)\}$$

$$= \frac{1}{N^2}\sum_n E\{c_0^2(n)\}E\{g^2(n)\}$$

$$= \frac{1}{N^2}\sum_n 1 \times 1$$

$$= \frac{1}{N^2}\sum_n 1 = \frac{1}{N^2}\cdot N = \frac{1}{N}$$

$$E\{r_{01}^2\} = \frac{1}{N}.$$

Multiuser Interference (MUI)

$$= h a_1 r_{01}$$

$$E\{|h|^2 |a_1|^2 |r_{01}|^2\}$$

$$= |h|^2 E\{|a_1|^2\} \cdot E\{|r_{01}|^2\}$$

$$= |h|^2 \cdot P_1 \times \frac{1}{N}$$

$$= |h|^2 P_1/N$$

Signal to Interference plus Noise Ratio, (SINR)

arises in context of CDMA

$$= \frac{|h|^2 P_0}{\frac{\sigma^2}{N} + \frac{|h|^2 P_1}{N}}$$

$$= \frac{N \times |h|^2 P_0}{\sigma^2 + |h|^2 P_1}$$

Spreading gain

MUI

$$SINR = \frac{|h|^2 P_0 N_0}{\sigma^2 + |h|^2 P_1}$$

Consider a scenario with $K+1$ users,

users $0, 1, \ldots, K+1$

$$SINR = \frac{N|h|^2 P_0}{\sigma^2 + |h|^2 \sum_{k=1}^{K} P_k}$$

Total power of all users other than user 0.

Multiuser CDMA Example:

- Consider a 2 user scenario with $P_0(dB) = P_1(dB) = 15\ dB$

$\sigma^2(dB) = 3\ dB$

$N = 256$ $\quad h = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$

What is SINR of user 0?

$$P_0(dB) = 15$$

$$10 \log_{10} P_0 = 15$$

$$\Rightarrow P_0 = 10^{1.5} = 31.62 = P_1$$

$$\sigma^2(dB) = 3 \Rightarrow \sigma^2 = 10^{0.3}$$

$$h = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j \Rightarrow |h|^2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$\approx 2$$

$$SINR = \frac{|h|^2 N \cdot P_0}{\sigma^2 + |h|^2 \cdot P_1}$$

$$= \frac{1 \times 256 \times 31.62}{3 + 1 \times 31.62}$$

$$= 233.81$$

$$SINR_{dB} = 10 \log_{10} 233.81$$

$$= 23.68 \text{ dB}.$$